Net current generation in a 1D quantum ring at zero magnetic field

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We study a non-adiabatic excitation of an electron system in a 1D quantum ring radiated by a short THz pulse. The response of two models, a continuous and discrete, is explored. By introducing a spatial asymmetry in the external perturbation a net current can be generated in the ring at a zero magnetic field. Effect of impurities and ratchets are investigated in combination with symmetric and asymmetric external excitation.

I. INTRODUCTION

Non-adiabatic generation of current in coherent quantum rings on the nanometer scale has attracted attention recently.[1,2,3] A one-dimensional (1D) quantum ring in its ground state exhibits a circulating current (persistent current) if the ring is placed in an external magnetic field perpendicular to the plane of the ring. The magnetic field breaks the left/right symmetry and the occupation of states carrying equal but opposite current is distorted. By subjecting the ring to a strong perturbation the persistent current can be changed non-adiabatically as the occupation of the single-electron states changes[1,4].

Here we consider two models of a one-dimensional ring, continuous and discrete, and investigate the response to a short-lived THz perturbation. The electrons confined to the ring are spinless and Coulomb interactions are neglected. The main objective is to see whether a current can be induced in the ring at a zero magnetic field by introducing a left/right asymmetry in the radiation pulse. The comparison between the two models shows identical results at least for the system parameters considered in the present work. We will also look at how impurities and ratchet-like potentials affect the resulting current.

II. THE MODEL

A. Continuous Model

We consider a 1D quantum ring of radius r_0 containing a few noninteracting, spinless electrons. The ring is described by the following unperturbed Hamiltonian in polar coordinates:

$$H_0 = -\frac{\hbar^2}{2m^* r_0^2}\frac{\partial^2}{\partial\theta^2} \tag{1}$$

where m^* is the effective mass of electrons. The Hamiltonian is symmetric under any rotation transformation. Consequently H_0 commutes with the angular momentum operator $L_z = -i\hbar\partial_\theta$. The common set of the eigenfunctions for the two commuting operators H_0 and L_z are

$$\psi_l(\theta) = \frac{e^{-il\theta}}{\sqrt{2\pi r_0}} \tag{2}$$

with $l = 0, \pm1, \pm2, ...$ The eigenvalue equations for the two operators are

$$H_0\psi_l(\theta) = E_l\psi_l(\theta) = \frac{\hbar^2 l^2}{2m^* r_0^2}\psi_l(\theta) \tag{3}$$

$$L_z\psi_l(\theta) = -l\hbar\psi_l(\theta). \tag{4}$$

Except for $l = 0$ all the states are doubly degenerate and carry a net persistent current that is normally quantified by the expectation value of the orbital magnetization operator

$$M_o = \frac{1}{2c}\mathbf{r}\times\mathbf{j} = -\frac{\mu_B}{\hbar}L_z. \tag{5}$$

B. Discrete Model

The discrete Hamiltonian of the 1D ring[5] is

$$\begin{aligned} H_0 = \quad & 2V\sum_n |n\rangle\langle n| \\ & -V\sum_n [\,|n\rangle\langle n+1| + |n\rangle\langle n-1|\,], \end{aligned} \tag{6}$$

and the angular momentum operator

$$L_z = \frac{\hbar}{i}\frac{N}{4\pi}\sum_n [\,|n\rangle\langle n+1| - |n\rangle\langle n-1|\,]. \tag{7}$$

The vectors $|n\rangle$ with $n = 1\cdots N$ define the N points of the ring of radius r_0 separated by a linear distance $a = 2\pi r_0/N$. The energy unit is $V = \hbar^2/(2ma^2)$. The common set of eigenvectors of the two commuting operators, H_0 and L_z, is

$$|\psi_l\rangle = \frac{1}{\sqrt{N}}\sum_n e^{-i\theta_n l}|n\rangle \tag{8}$$

with $l = 0, \pm1, \pm2, ..., \pm(N/2-1), N/2$ (for N even) and $\theta_n = \frac{2\pi n}{N}$ is the angle corresponding to the point n of the ring (in polar coordinates). The eigenvalue equations for the two operators are

$$H_0|\psi_l\rangle = E_l|\phi_l\rangle = (2V - 2V\cos(2\pi l/N))\,|\psi_l\rangle \tag{9}$$

$$L_z|\psi_l\rangle = -\frac{\hbar N}{2\pi}\sin(2\pi l/N)|\psi_l\rangle. \tag{10}$$

In the limit $l/N \to 0$ the eigenvalue set from the previous equations recover the values from the continuous model (eqs. (3) and (4)). If the ring contains impurities or ratchets, its Hamiltonian will be completed with the diagonal energies ϵ_n

$$H_0 \to H_0 + \sum_n \epsilon_n |n\rangle\langle n| \tag{11}$$

at chosen points on the ring. We define impurities by adding a diagonal energy $\epsilon_n = \epsilon_{imp}$ at three selected points $n = 1, 20, 40$. In case of ratchets[6,7], four equidistant saw-tooth potentials are placed at the points $n = 5,\ 15,\ 25,\ 35$ of the ring. The saw-tooth potential is defined by a series of three consecutive energies:

$$\epsilon_{n-1} = 0.1\epsilon_{rt},\ \epsilon_n = 0.2\epsilon_{rt},\ \epsilon_{n+1} = 0.3\epsilon_{rt}. \tag{12}$$

The net current in the discrete model is also calculated from the expectation value of the orbital magnetization (eq. (5)).

C. Time dependent perturbation

At $t = 0$ the quantum ring is radiated by a short THz pulse of duration $t_f \sim 10/\Gamma$ and frequencies ω_1, ω_2:

$$H_r(t) = V(\theta)\ W(t) \tag{13}$$
$$W(t) = e^{-\Gamma t} \sin(\omega_1 t) \sin(\omega_2 t). \tag{14}$$

The spatial component of the external pulse $V(\theta)$ is given by a combination of a dipole and a rotated quadrupole with amplitude A:

$$V(\theta) = A\left(\cos\theta + \cos 2(\theta + \phi_0)\right). \tag{15}$$

The deflection angle ϕ_0 between the dipole and quadrupole fields makes the external perturbation asymmetric for any values $\phi_0 \neq 0, \pi$. The perturbation is shown in Fig. 1. To follow the time-evolution of the system we use the density matrix to specify the state of the system. The ground-state density operator $\rho(t = 0)$ is constructed in terms of a given set of the eigenvectors $|\alpha\rangle$ of the initial time independent Hamiltonian

$$\rho(t = 0) = \sum_\alpha f(\epsilon_\alpha - \mu)\, |\alpha\rangle\langle\alpha|, \tag{16}$$

with the Fermi distribution f. The equation of motion for the density operator

$$i\hbar\partial_t \rho(t) = [H_0 + H_r(t), \rho(t)] \tag{17}$$

is inconvenient for numerical evaluation so we resort instead to the time-evolution operator T, defined by $\rho(t) = T(t)\rho(0)T^\dagger(t)$, leading to a simpler equation of motion

$$i\hbar\partial_t T(t) = H(t)T(t) \tag{18}$$

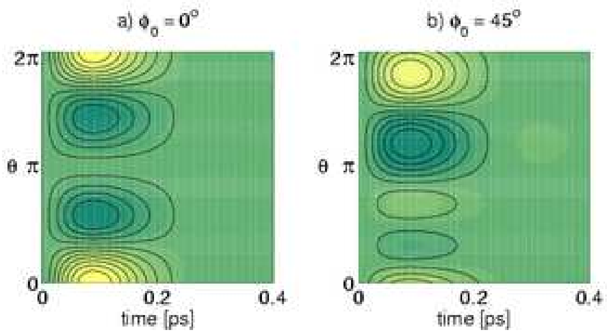

FIG. 1: (Color online) Spatial distribution of the external potential pulse as a function of time for (a) $\phi_0 = 0$ and (b) $\phi_0 = 45°$. The parameter values are listed in the caption of Fig. 2. Bright/yellow regions indicate maxima in the potential.

with the initial condition $T(0) = 1$. We solve the equation in the truncated basis of eigenvectors ψ_l (eq. (2)) for the continuous Hamiltonian (eq. (1)) or in the finite basis of eigenvectors of the discrete Hamiltonian (eqs. (6) or (11)) using the Crank-Nicholson scheme.[1] We calculate the time evolution of the system and extend especially the calculation of the induced current in the ring to times after the dependent perturbation vanishes. For quantifying the currents induced by $H_r(t)$ we calculate the expectation value of the orbital magnetization M_o (eq. 5) in terms of the density matrix

$$\mathcal{M}_o(t) = \mathrm{Tr}\left[M_0 \cdot \rho(t)\right]. \tag{19}$$

When the perturbation vanishes the time evolution of the system will be described by the time-evolution operator generated by the non-perturbed (and time independent) Hamiltonian H_0. The energy of the system will be constant

$$E(t > t_f) = \mathrm{Tr}\left[H_0 \cdot \rho(t)\right] = \textstyle\sum_l E_l \rho_{ll}(t_f), \tag{20}$$

and also the magnetization for a 1D pure ring

$$\mathcal{M}_o(t > t_f) = \mathrm{Tr}\left[M_o \cdot \rho(t)\right] = -\mu_B \textstyle\sum_l l\rho_{ll}(t_f). \tag{21}$$

We note that the non-zero values of the induced magnetization in Eq. (21) can be obtained only by different occupation of opposite current carrying states ψ_l and ψ_{-l} ($\rho_{ll} \neq \rho_{-l-l}$ at $t \simeq t_f$). On the other hand, the magnetization for a ring with a finite width will oscillate in time in the presence of a perpendicular magnetic field due to coupling of radial and angular modes of density oscillations, plasma oscillations.[1]

III. RESULTS

We use the effective mass of an electron in a GaAs, $m^* = 0.067m_e$. For modeling the ring with $n_e = 3$ electrons we set the radius to $r_0 = 14$nm and in the discrete

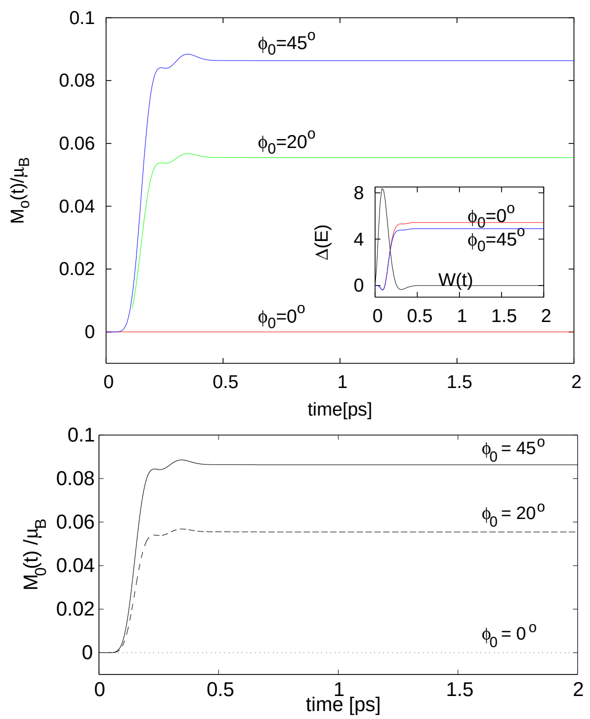

FIG. 2: (Color online) The time evolution of the orbital magnetization $\mathcal{M}_o(t)$ calculated with the discrete model (upper) and continuous model (lower) for the 1D ring described in the text. Three values of the deflection angle are presented, $\phi_0 = 0°, 20°, 45°$. The parameters of the external perturbation are $\hbar\omega_1 = 2.83$meV, $\hbar\omega_2 = 8.11$meV, $\hbar\Gamma = 4\hbar\omega_1 = 11.32$meV and $A = 67.68$meV. The frequencies are tuned to be comparable with the Bohr frequencies $\hbar\omega_{0,1} = 2.89$meV and $\hbar\omega_{1,2} = 8.6$meV. The inset presents the time evolution of the absorbed energy (in meV) of the discrete system for $\phi_0 = 0°, 45°$ and the time dependent part of the external perturbation W(t).

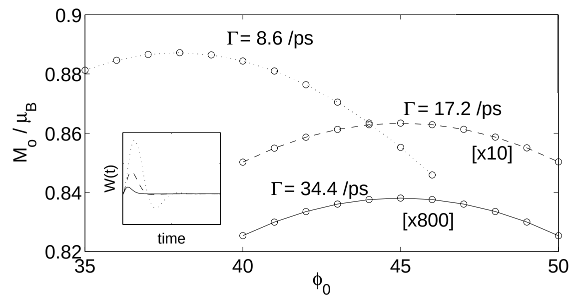

FIG. 3: The induced values of the orbital magnetization $\mathcal{M}_o$ vs. the deflection angle ϕ_0 for three different values of Γ (continuous model). The external perturbation parameters are the same as in Fig. 2. The scale of the two lower curves has been magnified as indicated. The inset depicts the time dependent part of the external perturbation for the three situations.

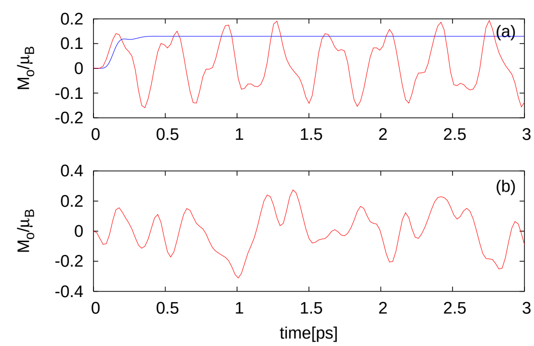

FIG. 4: (Color online) (a) The orbital magnetization $\mathcal{M}_o(t)$ as a function of time calculated with the discrete model of the 1D ring. The external perturbation is asymmetric ($\phi_0 = 45°$) with the same parameters as in Fig. 2 (except for A=135meV). The ring contains four ratchets with $\epsilon_{rt} = V$ (explained in text). The almost constant blue line is the time evolution of $\mathcal{M}_o(t)$ for a pure ring. (b) The ring with ratchets plus impurities present ($\epsilon_{imp} = 0.2V$). $V = \hbar^2/2m^*a^2 = 117.5$meV.

model use $N = 40$ points. Initially the system is in its ground state where right and left rotating states, $|\, l\,\rangle$ and $|-l\,\rangle$, are equally occupied and the current consequently zero ($\mathcal{M}_o(0) = 0$). In Fig. 2 we see that if the external pulse applied to the ring, like the ring itself, is left/right symmetric ($\phi_0 = 0, \pi$) a current can not be induced since the occupation of states remains symmetric. However, by introducing an asymmetry to the perturbation a constant current along the ring appears in both the continuous and discrete models.

For a weak excitation a perturbation analysis to 2nd order (in no external magnetic field) reveals that, for a single electron initially in the ground state $|\,0\,\rangle$, the difference between the transition probabilities to the states $|\,1\,\rangle$ and $|-1\,\rangle$ is proportional to $\sin 2\phi_0$

$$\mathcal{P}_{0,1}(t) - \mathcal{P}_{0,-1}(t) = F(t)\, \sin 2\phi_0 \qquad (22)$$

where $F(t)$ is a double integral depending on the parameters of the external perturbation. This implies that for a deflection angle of $\phi_0 = 45°$ the resulting current is at maximum. Fig. 3 shows the induced magnetization (continuous model) after the perturbation vanishes ($t > t_f$) as a function of the deflection angle ϕ_0 for three values of Γ. Clearly for strong excitation (lower Γ in Fig. 3) the results do not depend on ϕ_0 as eq. (22) shows, but have a more complex dependence on the exact form of the perturbation. As Γ decreases the maximum of the current shifts away from $\phi_0 = 45°$. The identical behavior is seen in the discrete model.

The constant magnitude of the current generated in the system by a left/right asymmetric excitation can be destroyed by the inclusion of ratchets and impurity potential as can be seen in Fig. 4. In Fig. 5 we see that the frequency of current oscillations depends heavily upon the strength of the ratchet potential. As the height of the ratchets increases the frequency grows. Furthermore the reduction of the current occurs even though the ratchet

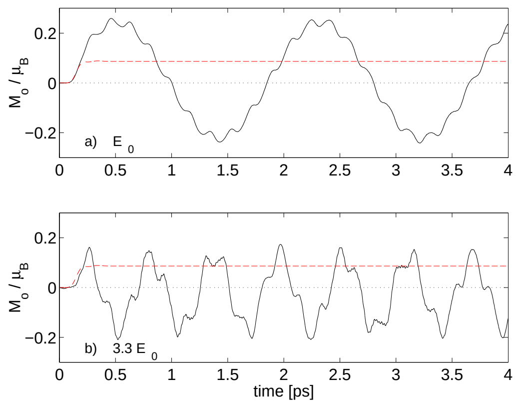

FIG. 5: (Color online) The orbital magnetization $\mathcal{M}_o(t)$ as a function of time calculated with the continuous model of the 1D ring. The external perturbation is asymmetric ($\phi_0 = 45°$) with the same parameters as in Fig. 2. The ring contains four ratchets (explained in text). The strength of ratchet potential $\epsilon_{rt} = E_0$ in (a) and $\epsilon_{rt} = 3.33E_0$ in (b). The dashed red curve is the time evolution of $\mathcal{M}_o(t)$ for a pure ring. $E_0 = \hbar^2/2m^* r_0^2 = 2.90$meV.

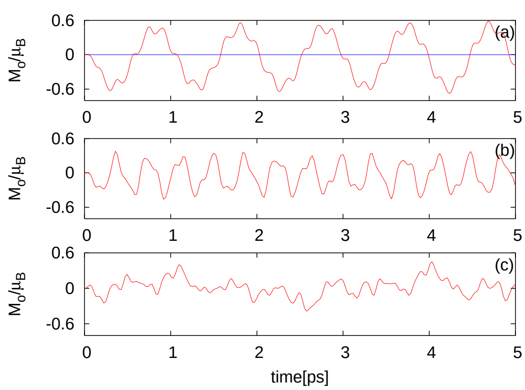

FIG. 6: (Color online) Oscillations of the orbital magnetization $\mathcal{M}_o(t)$ calculated with the discrete model. The ring is radiated with a symmetric pulse ($\phi_0 = 0°$). The perturbation parameters are the same as in Fig. 2 (except for A=135meV). (a) Four sticks symmetrically placed on the ring. Each stick is defined by adding a diagonal energy $\epsilon_n = 0.2V$ at points $n = 5, 15, 25, 35$ of the ring. The constant line is the time evolution of $\mathcal{M}_o(t)$ for a pure ring. (b) Four ratchets on the ring ($\epsilon_{rt} = V$). (c) Same as (b) plus impurities present ($\epsilon_{imp} = 0.2V$). $V = \hbar^2/2m^* a^2$.

potential is very weak. It is interesting that the period of oscillation can be much larger than that of the applied perturbation. The reader may now wonder if it would be possible to excite a current in the system by applying a left/right symmetric external electric field, but at the same time having an asymmetric static potential on the ring. We have tried several asymmetric potentials on the ring and different strength of the external excitation, but have not been successful in generating a steady DC current. Some results are seen in Fig. 6. Clearly, as we have drawn attention to above, the frequency of the current oscillations can be made very low compared to the dominant frequencies included in the excitation pulse.

IV. CONCLUSIONS

For a quantum ring in no external magnetic field a left/right asymmetric excitation is essential for the appearance of a circulating current. In an external magnetic field the energy of states whose orbital angular momentum is aligned with the field is lowered, thus breaking the symmetry and generating a DC current. Here we have seen that a similar effect can be accomplished by an asymmetric excitation pulse at zero magnetic field, but a DC current can not be generated by a left/right symmetric excitation even though the ring itself is made asymmetric by a static potential or by change to the geometry, breaking the perfect circular symmetry. The introduction of a ratchet potential in the ring that is excited by an asymmetric external field changes the DC current created into an AC current with arbitrary low frequency depending on the intensity of the ratchet. This is a clear nonlinear behavior that could not be observed by traditional linear response methods.

The electron-electron interaction has been neglected here as the effects studied do not depend on it qualitatively, but certainly quantitatively they do.[8]

Acknowledgments

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